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RESEARCH ARTICLE

Some Abstract Properties of Semigroups Appearing in Superconformal Theories

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Abstract

A new type of semigroup which appears while dealing with $N = 1$ superconformal symmetry in superstring theories is considered. The ideal series having unusual abstract properties is constructed. Various idealisers are introduced and studied. The ideal quasicharacter is defined. Green's relations are found and their connection with the ideal quasicharacter is established.

1. Introduction

Mathematical objects with new properties often appear from concrete physical considerations and models. The discovery of supersymmetry [36, 37] gave many new mathematical features, but its influence on the general abstract properties of the theory, in spite of the fact that among principal objects there were noninvertible ones and zero divisors [16], needs to be emphasized. The latter leaded to the conclusion that the abstract ground of supersymmetric theory should have a semigroup nature [8]. It was also realised that the noninvertible transformations and semigroups appearing in that way have many new nontrivial properties [7, 10]. In particular, it would be interesting to work out the general abstract structure of the $N = 1$ superconformal semigroup, which is important in the consistent construction of the superstring unified theories [17, 12]. In this paper we provide a consideration of the superconformal semigroups from the abstract-algebraic point of view and present their abstract properties without proofs which will appear elsewhere.

2. Preliminaries

The semigroup of $N = 1$ superconformal transformations of $C^{1,1}$ complex superspace with the coordinates (z, θ) valued in the Grassmann algebra [2], where $z \in C^{1,0}$ and $\theta \in C^{0,1}$, is isomorphic to the semigroup S of the even $C^{1,0} \to C^{1,0}$ and odd $C^{1,0} \to C^{0,1}$ functions satisfying some multiplication law (for details see [7, 9]). The even part of the law

$$\mathrm{s}_3 = \mathrm{s}_1 * \mathrm{s}_2, \quad \mathrm{s}_i \in \mathrm{S}, \tag{1}$$

in terms of the even functions $g(z)$ can be presented as

$$g_3(z) = [g_1(\tilde{z}) + h_1(\tilde{z})] \cdot g_2(z), \tag{2}$$

where $\tilde{z}$ is some shifting and $h_1(z)$ is some even nilpotent function of second degree, i.e. $h_1^2(z) \equiv h_1(z) \cdot h_1(z) = 0$. We stress that, because of the shifting $z \to \tilde{z}$ and

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the second term in the brackets (2), $\mathbf{S}$ differs from the semigroups of functions with point by point multiplication [6], and also from the semigroups of functions [24, 25]. This leads to new unusual abstract properties of $\mathbf{S}$ considered below. Further we note that to study these properties it is sufficient to know the formal expression (2) only. This parametrisation of $N = 1$ superconformal transformations was given in [7, 9] (where one can also find the exact formulas and the concrete background). For other considerations we refer to [1, 27, 28, 15].

Here we do not consider the physical interpretations of $g(z)$ (see [1, 5]) and stress only that $g(z)$ controls invertibility of the superconformal transformations [8]. Therefore, the index of $g(z)$ which is defined by

$$\operatorname{ind} g(z) \overset{def}{=} \left\{n \in \mathbb{Z} \mid g^n(z) = 0, g^{n-1}(z) \neq 0\right\} \tag{3}$$

plays a crucial part in the following. We mention here that in (2) and (3) the multiplication is a point by point one in the Grassmann algebra [2] (for clarity sometimes we use a point for it), but the star in (1) denotes the semigroup multiplication.

So the semigroup $\mathbf{S}$ can be divided into two disjoint parts $\mathbf{S} = \mathbf{G} \cup \mathbf{T}$, $\mathbf{G} \cap \mathbf{T} = \emptyset$, where

$$\mathbf{G} \overset{def}{=} \{\mathbf{s} \in \mathbf{S} \mid \operatorname{ind} g(z) = \infty\}, \tag{4}$$

$$\mathbf{T} \overset{def}{=} \{\mathbf{s} \in \mathbf{S} \mid \operatorname{ind} g(z) < \infty\}. \tag{5}$$

Here $\mathbf{G}$ is a group corresponding to the invertible transformations. From the multiplication law (2) it follows that $\mathbf{T}$ is a two-sided ideal. The unity element $\mathbf{e} \in \mathbf{S}$ has $g(z) = 1$, $h(z) = 0$, and the zero element has $g(z) = 0$, $h(z) = 0$ (for other details see [8, 9]). From (2) and the relation $\operatorname{ind} h(z) = 2$ it follows that $\mathbf{T}$ is a nilsemigroup [21, 13, 11, 34], i.e. $\forall \mathbf{t} \in \mathbf{T}$ $\exists n \in \mathbb{Z}$, $\mathbf{t}^{*n} = \mathbf{z}$ (here the multiplication in the power expression is implied as the semigroup one (1)). So every element from $\mathbf{T}$ is nilpotent without bound on its index and of finite order, but every element from $\mathbf{G}$ is of infinite order.

The superconformal transformations corresponding to $\mathbf{G}$ were studied earlier in [1, 5, 29]. Therefore we concentrate our attention on the ideal $\mathbf{T}$, which gives the evidence of some unusual abstract properties of the parametrised superconformal semigroup $\mathbf{S}$.

3. Ideal Series

To classify the elements from the ideal part $\mathbf{T}$ we take the n-th power of the equation (2) in the Grassmann algebra and, using the relation $\operatorname{ind} h(z) = 2$, obtain

$$g_3^n(z) = \left[g_1^n(\tilde{z}) + n \cdot g_1^{n-1}(\tilde{z}) \cdot h_1(\tilde{z})\right] \cdot g_2^n(z). \tag{6}$$

We see that the natural classification can be done by means of the index of $g(z)$ (see (3)). Let us define the following sets

$$\mathbf{\Delta}_n \overset{def}{=} \{\mathbf{s} \in \mathbf{S} \mid \operatorname{ind} g(z) = n\}. \tag{7}$$

$$\mathbf{I}_n \overset{def}{=} \bigcup_{k \leq n} \mathbf{\Delta}_k. \tag{8}$$

Then we notice that $\mathbf{T}$ is a disjoint union of the sets $\mathbf{\Delta}_n$, because $\mathbf{T} = \bigcup_n \mathbf{\Delta}_n$, $\mathbf{\Delta}_n \cap \mathbf{\Delta}_{n-1} = \emptyset$. From (6) it follows that $\mathbf{I}_{n-1} \subset \mathbf{I}_n$ and $\mathbf{I}_n \setminus \mathbf{I}_{n-1} = \mathbf{\Delta}_n$. Therefore we obtain the following infinite chain of the sets $\mathbf{I}_n$

$$\mathbf{z} \subset \mathbf{I}_1 \subset \mathbf{I}_2 \subset \cdots \subset \mathbf{I}_n \subset \cdots \subset \mathbf{T}. \tag{9}$$

To understand the meaning of $\mathbf{I}_n$ we use (6) and obtain

$$\mathbf{S} * \mathbf{I}_n \subseteq \mathbf{I}_n, \tag{10}$$

$$\mathbf{I}_n * \mathbf{S} \subseteq \mathbf{I}_{n+1}, \tag{11}$$

$$\mathbf{S} * \mathbf{I}_n * \mathbf{S} \subseteq \mathbf{I}_{n+1}, \tag{12}$$

From these relations we can easily observe that the sets $\mathbf{I}_n$ are left ideals of the semigroup $\mathbf{S}$, but not right ideals, because of (11). Moreover, the appearance of $n+1$ in the right side of (11) and (12) is very unusual, and so is natural to call these strange sets $\mathbf{I}_n$ "jumping ideals". Therefore $\mathbf{I}_{n-1} \lhd_l \mathbf{I}_n$ and the chain (9) is a left and "jumping ideal" series. Then $\mathbf{I}_n$ are quasiideals [33, 4] since they satisfy $\mathbf{S} * \mathbf{I}_n \cap \mathbf{I}_n * \mathbf{S} \subseteq \mathbf{I}_n$. Simultaneously, the sets $\mathbf{I}_n$ are biideals, because $\mathbf{I}_n * \mathbf{S} * \mathbf{I}_n \subseteq \mathbf{I}_n$ [3, 20]. It is exciting that in our case the regularity is not necessary for the coincidence of quasiideals and biideals in superconformal semigroup (as distinct from [20]). Because of the inclusion $\mathbf{I}_n \lhd \mathbf{U} \Rightarrow \mathbf{I}_n \lhd \mathbf{S}$, $\forall \mathbf{U} \lhd \mathbf{S}$ the semigroup $\mathbf{S}$ is a filial semigroup [18]. The indices in (9) form a well ordered set for which n is an ordinal. Because of $\mathbf{I}_{n-1} \lhd_l \mathbf{I}_n$ the chain (9) can be called a left ascending ideal series of $\mathbf{S}$. From (11) and (12) we derive

$$\mathbf{S} * \mathbf{I}_n \cup \mathbf{I}_n * \mathbf{S} \subseteq \mathbf{I}_{n+1}, \tag{13}$$

This condition is opposite to that for which the chain (9) is an ascending annihilator series of $\mathbf{S}$ [14, 31]. So we call it an ascending antiannihilator series of $\mathbf{S}$.

The multiplication law for the sets $\mathbf{I}_n$ and $\mathbf{\Delta}_n$ is

$$\begin{array}{lcl}
\mathbf{I}_n * \mathbf{I}_{n+k} & \subseteq & \mathbf{I}_{n+1}, \\
\mathbf{I}_{n+k-1} * \mathbf{I}_n & \subseteq & \mathbf{I}_n, \\
\mathbf{\Delta}_n * \mathbf{\Delta}_{n+k} & \subseteq & \mathbf{I}_{n+1}, \\
\mathbf{\Delta}_{n+k-1} * \mathbf{\Delta}_n & \subseteq & \mathbf{I}_n, \\
\mathbf{I}_n * \mathbf{\Delta}_{n+k} & \subseteq & \mathbf{I}_{n+1}, \\
\mathbf{I}_{n+k-1} * \mathbf{\Delta}_n & \subseteq & \mathbf{I}_n, \\
\mathbf{\Delta}_n * \mathbf{I}_{n+k} & \subseteq & \mathbf{I}_{n+1}, \\
\mathbf{\Delta}_{n+k-1} * \mathbf{I}_n & \subseteq & \mathbf{I}_n, \\
\mathbf{I}_n * \mathbf{G} & \subseteq & \mathbf{I}_{n+1}, \\
\mathbf{G} * \mathbf{I}_n & \subseteq & \mathbf{I}_n, \\
\mathbf{\Delta}_n * \mathbf{G} & \subseteq & \mathbf{I}_{n+1}, \\
\mathbf{G} * \mathbf{\Delta}_n & \subseteq & \mathbf{\Delta}_n.
\end{array} \tag{14}$$

where $k > 0$. It follows that the set $\mathbf{I}_n$ is a subsemigroup of $\mathbf{S}$, because from (14) we have $\mathbf{I}_n * \mathbf{I}_n \subseteq \mathbf{I}_n$ but the set $\mathbf{\Delta}_n$ is not a subsemigroup, since $\mathbf{\Delta}_n * \mathbf{\Delta}_n \subseteq \mathbf{I}_n$. This is a consequence of the fact that our semigroup is defined over the Grassmann algebra [2] which contains nilpotents and zero divisors, and the latter fact should be taken into account properly [16].

From the last two relations of (14) and (12) we can obtain

$$\mathbf{G} * \mathbf{\Delta}_n * \mathbf{G} \subseteq \mathbf{I}_{n+1}, \tag{15}$$

i.e. some of the elements from $\mathbf{\Delta}_n$ are conjugated by the subgroup $\mathbf{G}$ with the elements of the next set $\mathbf{\Delta}_{n+1}$. By analogy with [35, 23, 22] we define G-normal subsets $\mathbf{A}, \mathbf{B} \subseteq \mathbf{S}$ as follows: $\mathbf{g}^{-1} * \mathbf{A} * \mathbf{g} \subseteq \mathbf{B}$, $\mathbf{g} \in \mathbf{G}$. Then from (15) we make a conclusion that any two sets $\mathbf{\Delta}_n$ contain G-normal elements and one can reach any $\mathbf{\Delta}_n$ using the subgroup action only. Further general abstract properties of such elements can be found in [23, 30].

4. Idealisers

The left (right, two-sided) idealiser $I_l(\mathbf{U})$ ($I_r(\mathbf{U})$, $I(\mathbf{U})$) of the subset $\mathbf{U} \subseteq \mathbf{S}$ can be defined as the largest subsemigroup of $\mathbf{S}$ within which $\mathbf{U}$ is a left (right, two-sided) ideal, i.e.

$$I_l(\mathbf{U}) \overset{def}{=} \{\mathbf{s} \subseteq \mathbf{S} \mid \mathbf{s} * \mathbf{U} \subseteq \mathbf{U}\}, \tag{16}$$

$$I_r(\mathbf{U}) \overset{def}{=} \{\mathbf{s} \subseteq \mathbf{S} \mid \mathbf{U} * \mathbf{s} \subseteq \mathbf{U}\}, \tag{17}$$

$$I(\mathbf{U}) \overset{def}{=} \{\mathbf{s} \subseteq \mathbf{S} \mid \mathbf{s} * \mathbf{U} \subseteq \mathbf{U},\ \mathbf{U} * \mathbf{s} \subseteq \mathbf{U}\}. \tag{18}$$

The set $I(\mathbf{U})$ is a subsemigroup, since from

$$\mathbf{U} * \mathbf{s} \subseteq \mathbf{U},\ \mathbf{s} * \mathbf{U} \subseteq \mathbf{U},\ \mathbf{U} * \mathbf{t} \subseteq \mathbf{U} \text{ and } \mathbf{t} * \mathbf{U} \subseteq \mathbf{U}$$

we may deduce that

$$\mathbf{U} * \mathbf{s} * \mathbf{t} \subseteq \mathbf{U} * \mathbf{t} \subseteq \mathbf{U} \text{ and } \mathbf{s} * \mathbf{t} * \mathbf{U} \subseteq \mathbf{s} * \mathbf{U} \subseteq \mathbf{U}.$$

Also, if $\mathbf{V}$ is a subsemigroup of $\mathbf{U}$ and $\mathbf{U} \triangleleft \mathbf{V}$, then $\mathbf{v} * \mathbf{U} \subseteq \mathbf{U}$, $\mathbf{U} * \mathbf{v} \subseteq \mathbf{U}$ for all $\mathbf{U}$ in $\mathbf{V}$ and so $\mathbf{V} \subseteq I(\mathbf{U})$.

Let us consider the idealisers of the various introduced subsets of $\mathbf{S}$. First the left idealiser for $\mathbf{I}_n$ is $\mathbf{S}$, as follows directly from (10), i.e.

$$I_l(\mathbf{I}_n) = \mathbf{S}. \tag{19}$$

From the last relation in (14) we find

$$I_l(\mathbf{\Delta}_n) = \mathbf{G}. \tag{20}$$

For the right idealisers of $\mathbf{I}_n$ the situation is more complicated. Using (11) we divide $\mathbf{S}$ into two disjoint parts $\mathbf{S} = \mathbf{S^I}_n \cup \mathbf{S^\Delta}_n$, where $\mathbf{S^I}_n \cap \mathbf{S^\Delta}_n = \emptyset$, and they satisfy the relations

$$\mathbf{I}_n * \mathbf{S^I}_n \subseteq \mathbf{I}_n\ . \tag{21}$$

$$\mathbf{I}_n * \mathbf{S^\Delta}_n \subseteq \mathbf{\Delta}_{n+1}\ . \tag{22}$$

By definition (17) $\mathbf{S^I}_n$ is the right idealiser for $\mathbf{I}_n$, i.e.

$$I_r(\mathbf{I}_n) = \mathbf{S^I}_n. \tag{23}$$

Obviously $\mathbf{I}_n \subset \mathbf{S^I}_n$, since $\mathbf{I}_n * \mathbf{I}_n \subset \mathbf{I}_n$. Therefore $\mathbf{S^I}_n = \mathbf{I}_n \cup \mathbf{S^{II}}_n$. From (6) it follows that for the elements from $\mathbf{S^{II}}_n$ the second term in the brackets should disappear, and therefore we find

$$\mathbf{S^{II}}_n = \left\{ \mathrm{s} \in \mathbf{T} \setminus \mathbf{I}_n \mid g_1^{n-1}(\tilde{z}) \cdot g_2^n(z) = 0,\ h_1(\tilde{z}) \cdot g_2^n(z) = 0 \right\}. \tag{24}$$

Then the "jumping" set $\mathbf{S^\Delta}_n$ from (22) is equal to $\mathbf{S^\Delta}_n = (\mathbf{S} \setminus \mathbf{I}_n) \setminus \mathbf{S^{II}}_n$.

Another way to vanish the second term in (6) is by considering the special superconformal transformations (they are called *Ann*-transformations in [9]) for which the relation $g^{n-1}(z) \cdot h(z) = 0$ is valid (see (2) and (6)). Let us divide $\mathbf{I}_n$ in two disjoint parts $\mathbf{I}_n = \mathbf{I^A}_n \cup \mathbf{I^{\neq A}}_n$, where $\mathbf{I^A}_n \stackrel{def}{=} \{\mathrm{s} \in \mathbf{I}_n \mid g^{n-1}(z) \cdot h(z) = 0\}$. It was shown in [9] that the *Ann*-property is preserved from the right only, and so we obtain $\mathbf{I^A}_n * \mathbf{S} \subseteq \mathbf{I^A}_n$, which means that $\mathbf{I^A}_n$ is a right ideal in $\mathbf{S}$, then

$$I_r\left(\mathbf{I^A}_n\right) = \mathbf{S}. \tag{25}$$

For the sets $\mathbf{\Delta^A}_n = \mathbf{I^A}_n \setminus \mathbf{I^A}_{n-1}$ we find $\mathbf{\Delta^A}_n * \mathbf{G} \subseteq \mathbf{\Delta^A}_n$, and therefore

$$I_r\left(\mathbf{\Delta^A}_n\right) = \mathbf{G}. \tag{26}$$

We note here that by means of the right group action we can reach a set $\mathbf{I}_n$ with any large n, because the relation $\mathbf{\Delta^{\neq A}}_n * \mathbf{G} \subseteq \mathbf{\Delta^{\neq A}}_{n+1}$ (see also (15)).

5. Ideal Quasicharacter

Let us define

$$\chi(\mathrm{s}) \stackrel{def}{=} \{n \in \mathbb{N} \mid \operatorname{ind} g(z) = n\}. \tag{27}$$

Using (10) and (11) we obtain

$$\max \chi(\mathrm{s} * \mathbf{t}) = \begin{cases} \chi(\mathbf{t}), & \chi(\mathrm{s}) \geq \chi(\mathbf{t}) \\ \chi(\mathrm{s}) + 1, & \chi(\mathrm{s}) < \chi(\mathbf{t}). \end{cases} \tag{28}$$

In particular,

$$\begin{aligned} \chi(\mathbf{g} * \mathrm{s}) &= \chi(\mathrm{s}), \\ \chi(\mathrm{s} * \mathbf{g}) &= \chi(\mathrm{s}) + 1, \end{aligned} \quad \mathrm{s} \neq \mathbf{z}. \tag{29}$$

From (28) it follows that $n_\delta = |\chi(\mathrm{s} * \mathbf{t}) - \chi(\mathrm{s}) - \chi(\mathbf{t})|$ is bounded. This value n_δ shows how much the mapping $\mathrm{s} \to \chi(\mathrm{s})$ differs from a homomorphism [19] . The boundedness of n_δ allows us to conclude that $\chi(\mathrm{s})$ is a quasicharacter [32] which can be called an ideal quasicharacter. The elements of $\mathbf{S}$ having finite ideal quasicharacter are nilpotent and belong to the ideal $\mathbf{T}$, and $\chi(\mathbf{g}) = \infty$, $\mathbf{g} \in \mathbf{G}$. Another description of the ideal quasicharacter can be written as follows $\chi(\mathrm{s}) = n \Longleftrightarrow \mathrm{s} \in \mathbf{\Delta}_n$. Since $\mathbf{\Delta}_n \cap \mathbf{\Delta}_m = \emptyset$, $n \neq m$, we conclude that $\chi(\mathrm{s})$ indeed separates the elements having different indices, and the relation π defined as $\mathrm{s}\pi t \Longleftrightarrow \chi(\mathrm{s}) = \chi(\mathbf{t})$ is an equivalence relation in $\mathbf{S}$.

6. Green's Relations

In our notations the Green's $\mathcal{L}$ and $\mathcal{R}$ relations are

$$\begin{aligned} \mathbf{s}\,\mathcal{L}\,\mathbf{t} &\iff \exists \mathbf{u},\mathbf{v}\in\mathbf{S},\ \ \mathbf{u}*\mathbf{s}=\mathbf{t},\ \ \mathbf{v}*\mathbf{t}=\mathbf{s}, \\ \mathbf{s}\,\mathcal{R}\,\mathbf{t} &\iff \exists \mathbf{u},\mathbf{v}\in\mathbf{S},\ \ \mathbf{s}*\mathbf{u}=\mathbf{t},\ \ \mathbf{t}*\mathbf{v}=\mathbf{s}. \end{aligned} \tag{30}$$

Let us find $\mathcal{L}$ and $\mathcal{R}$ equivalent elements in the superconformal semigroup $\mathbf{S}$. Using (10) and (28) we find that $\mathbf{s}\,\mathcal{L}\,\mathbf{t} \Rightarrow \chi(\mathbf{s}) \le \chi(\mathbf{t}) \wedge \chi(\mathbf{t}) \le \chi(\mathbf{s}) \Rightarrow \chi(\mathbf{s}) = \chi(\mathbf{t})$. Therefore $\mathcal{L} = \pi$, and $\mathcal{L}$-equivalent elements have the same ideal quasicharacter,

$$\mathbf{s}\,\mathcal{L}\,\mathbf{t} \Rightarrow \chi(\mathbf{s}) = \chi(\mathbf{t}), \tag{31}$$

and they belong to the same set $\mathbf{\Delta}_n$. By analogy from (11) for the $\mathcal{R}$-equivalent elements we derive $\mathbf{s}\,\mathcal{R}\,\mathbf{t} \Rightarrow \chi(\mathbf{s}) \le \chi(\mathbf{t})+1 \wedge \chi(\mathbf{t}) \le \chi(\mathbf{s})+1$. Then the ideal quasicharacters of the $\mathcal{R}$-equivalent elements can differ only by 1 or coincide, i.e.

$$\mathbf{s}\,\mathcal{R}\,\mathbf{t} \Rightarrow |\chi(\mathbf{s}) - \chi(\mathbf{t})| \le 1. \tag{32}$$

Since $\mathcal{H} = \mathcal{L}\cap\mathcal{R}$, the sets $\mathbf{\Delta}_n$ consist also of $\mathcal{H}$-equivalent elements.

Consider the $\mathcal{L}$-equivalent elements. Let $\mathbf{s}\neq\mathbf{t}, \mathbf{s}\neq\mathbf{z}, \mathbf{t}\neq\mathbf{z}$. From (30) we derive that $\mathbf{s} = \mathbf{v}*(\mathbf{u}*\mathbf{s}) = (\mathbf{v}*\mathbf{u})*\mathbf{s} = (\mathbf{v}*\mathbf{u})^{*k}*\mathbf{s}$ for any $k\in\mathbb{N}$. If $\mathbf{v}\in\mathbf{T}\vee\mathbf{u}\in\mathbf{T}$, then $(\mathbf{v}*\mathbf{u})^{*k}\in\mathbf{T}$, since $\mathbf{T}$ is an ideal in $\mathbf{S}$. Because of $\mathbf{T}$ is a nilsemigroup $\exists n\in\mathbb{N}$ such that $(\mathbf{v}*\mathbf{u})^{*n} = \mathbf{z}$. Through the arbitrariness of k we choose $k=n$ and obtain $\mathbf{s} = (\mathbf{v}*\mathbf{u})^{*n}*\mathbf{s} = \mathbf{z}*\mathbf{s} = \mathbf{z}$ or $\mathbf{s}=\mathbf{t}$, which contradicts the initial assumptions. The same is valid for other Green's relations. Therefore $\mathbf{v}\in\mathbf{G}\wedge\mathbf{u}\in\mathbf{G}$, i.e. nontrivial $\mathcal{L}$ and $\mathcal{R}$ equivalences can be constructed with regard to the invertible elements of $\mathbf{S}$ only. In fact the principal left and right ideals generated by $\forall\mathbf{t}\in\mathbf{S}$ and defined by $\mathbf{L}(\mathbf{t}) \overset{def}{=} \mathbf{S}*\mathbf{t}$ and $\mathbf{R}(\mathbf{t}) \overset{def}{=} \mathbf{t}*\mathbf{S}$ are analogous to the left and right cosets of $\mathbf{G}$ in $\mathbf{S}$ introduced in [26, 30].

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